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A QUASI PROOF OF THE ARITHMETICAL MEAN.

BY ORMOND STONE.

GIVEN any number of observations made under precisely similar circumstances, for the determination of a single unknown quantity, and let it be assumed that the most probable value which can be obtained from two such observations will be the arithmetical mean of the individual determinations; to prove that the most probable value which can be obtained from any number of such observations is also the arithmetical mean.

Suppose three values a, b, c given by observations made under similar circumstances. By hypothesis,

$$\tfrac{1}{2}(a+b),\ \tfrac{1}{2}(b+c),\ \tfrac{1}{2}(c+a)$$

are the most probable values which can be obtained from any two of the observations.

Each of these results may in turn be considered as a direct observation made under more favorable circumstances, from which we can again obtain the most probable value taking them two and two; viz.:

$$\tfrac{1}{4}(a+2b+c),\ \tfrac{1}{4}(a+b+2c),\ \tfrac{1}{4}(2a+b+c).$$

Combining these in a similar manner, we have

$$\tfrac{1}{8}(2a+3b+3c),\ \tfrac{1}{8}(3a+2b+3c),\ \tfrac{1}{8}(3a+3b+2c);$$

or, in general, for any even number n of such combinations,

$$\frac{1}{2}\left[\frac{2^n-1}{3}a+\left(\frac{2^n-1}{3}+1\right)b+\frac{2^n-1}{3}c\right]=\frac{1}{3}\left[\left(1-\frac{1}{2^n}\right)a+\left(1+\frac{1}{2^{n-1}}\right)b\right.$$
$$\left.+\left(1-\frac{1}{2^n}\right)c\right],$$

etc.;

and for any odd number,

$$\frac{1}{2^n}\left[\left(\frac{2^n+1}{3}-1\right)a+\frac{2^n+1}{3}b+\frac{2^n+1}{3}c\right]=\frac{1}{3}\left[\left(1-\frac{1}{2^{n-1}}\right)a+\left(1+\frac{1}{2^n}\right)b\right.$$
$$\left.+\left(1+\frac{1}{2^n}\right)c\right],$$

etc.

If n be infinite the corresponding values will evidently be the best that can be obtained upon the hypothesis which forms the basis of this investigation. For $n=\infty$, however, the three values are each equal to

$$\tfrac{1}{3}(a+b+c).$$

In a similar manner it can be shown that if the arithmetical mean is the most probable value for any number of observations p, the same will be true for the number $p+1$. Hence, the proposition is true for any number whatever.

CORRESPONDENCE.—EDITOR ANALYST: Permit me to say thro' your columns that I do not wish to be considered responsible for the corrections appended by Prof Wood, p. 128, to his article on quaternions in the last ANALYST. I have heretofore sent, either to Prof. W. directly, or to the Editor, criticisms on what I deemed errors in these papers on quaternions, viz., on the following:

p. 36, §9.

p. 67, the last two lines.

p. 68, the transformation $kk = \frac{j}{i} . \frac{-i}{j} = \frac{-i}{i}$.

p. 69, the sentence beginning on line 7.

p. 70, "The eq. $k^2 = -1$, when k has a fixed position, is the equation of a circumference".

p. 71, line 16. And I would now respectfully call Prof. Wood's attention to the following errors, &c., in the last article:

p. 123, two sentences beginning line 11 from bottom.

p. 128, "when the direction of rotation of the fraction and its reciprocal are in opposite senses".

p. 128, line 6, the reason assigned for the correction.

ALEX. S. CHRISTIE.

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[As the editor does not profess to be *read* in the science of Quaternions, and as he supposes many of his readers are in the same condition, he has hitherto admitted to the pages of the ANALYST only papers, on that science, of an elementary character and by responsible authors. Mr. Christie's criticisms, above referred to, were therefore, with his permission, sent to Prof. Wood with the hope that any material defect in the papers of Prof. Wood might be corrected without further discussion.

As Mr. Christie's criticisms were not returned to us by Prof. Wood, we are not able to insert them here that our readers might judge whether they have been properly responded to by Prof. Wood, and as the subject is one upon which clear and accurate knowledge is important, we hereby tender to Mr. Christie space in our next issue for an exposition of the passages above cited.—Ed.]